

Modeling of Frequency Dependent Losses of Transmission Lines with VHDL-AMS in Time Domain

Kerstin Siebert^{#1}, Harald Günther^{#2}, Stephan Frei^{#3}, Wolfgang Mickisch^{*2}

[#] *Arbeitsgebiet Bordsysteme, Technische Universität Dortmund
Friedrich-Wöhler-Weg 4, 44227 Dortmund, Germany*

¹ kerstin.siebert@tu-dortmund.de ² harald.guenther@tu-dortmund.de ³ stephan.frei@tu-dortmund.de

^{*} *TÜV Nord*

Adlerstr. 7, 45307 Essen, Germany

² wmickisch@tuev-nord.de

Abstract—A time domain model of a three conductor transmission line considering frequency depended losses, e.g. skin-effect, for the modeling language VHDL-AMS was developed. The calculation features of VHDL-AMS were considered and flexible and efficient approximations for the frequency dependent propagation and admittance functions could be implemented. The model can be used for linear and non-linear time and frequency domain simulations. It forms a basis for EMC extensions that can be implemented using the developed approximation techniques. Modern standardized modeling languages like VHDL-AMS (Very High Speed Integrated Circuit Hardware Description Language – Analog and Mixed Signal) have the important advantage that growing model libraries permit fast creation of complex simulation models. Exchange and extension of models is easily possible. After a short introduction and presentation of the theory used for modeling, application examples are shown. The developed model is compared to measurement results and to a lossless multiconductor transmission line model. The validity of the implemented VHDL-AMS model is proved.

I. INTRODUCTION

Differential three conductor transmission lines (TL) play an important role in many technical areas e.g. for automobile bus systems like FlexRay. Here simple and cheap TLs are used even for safety critical applications. Reliable data transmission also at very high data rates implicating very high frequencies is essential. Using simulation techniques statements about the signal integrity and EMC can already be made in early phases of development.

For accurate simulation results at high data rates the frequency dependent losses of a MTL have to be modeled. [1], [2] and [3] give extensive descriptions on different approaches to model TLs including skin effect in time domain. One approach is to use FDTD to model the skin effect. In [4] a VHDL-AMS MTL implementation based on FDTD is described.

In [5] and [6] implementation algorithms are proposed that were used already in the circuit simulator HSpice. A similar approach was applied here for the creation of the model with the modeling language VHDL-AMS. The given approach

implements analytical approximations of the attenuation and the characteristic admittance functions directly in VHDL-AMS.

VHDL-AMS [7] was selected as modeling language to make the model suitable for practical applications. The VHDL-AMS modeling language became within the last years more and more famous in many industrial branches. It can be easily edited and extended. Models can be encapsulated as devices interfacing with their environment only via well defined ports. This makes it very easy to use the TL-VHDL-AMS-model in combination with other models like bus transceivers. Complex structures of realistic automotive cable configurations can be built and investigated.

II. MODELLING OF FREQUENCY DEPENDENT LOSSES

A Transmission Line (TL) can be described by the following equations [1]:

$$I_1(s) = Y_0(s)V_1(s) - (Y_0(s)V_2(s) - I_2(s))e^{-\gamma(s)l} \quad (1)$$

$$I_2(s) = -Y_0(s)V_2(s) + (Y_0(s)V_1(s) + I_1(s))e^{-\gamma(s)l} \quad (2)$$

The frequency dependency of the characteristic admittance

$$Y_0(s) = \sqrt{\frac{G'+sC'}{R'+sL'}} \quad (3)$$

and of the propagation function $W(s) = e^{-\gamma(s)l}$ with

$$\gamma(s) = \sqrt{(R'(s) + sL')(G' + sC')} = s\sqrt{L'C'} \sqrt{\left(1 + \frac{R'}{sL'}\right)\left(1 + \frac{G'}{sC'}\right)} \quad (4)$$

and l as the length of the TL has to be considered.

A. Treatment of the Propagation Function

The propagation function $W(s) = e^{-\gamma(s)l}$ depends on the frequency and can not be implemented directly in time domain solvers like VHDL-AMS. State space or transfer function representations are required that can be transferred easily into a set of differential equations that can be calculated by the integrated DAE solvers. A transfer function described by a

rational function approximated using Padé approximation [8] for example can give a direct solution implementable in VHDL-AMS using the 'LTF attribute.

$$e^{-\gamma(s)l} = W(s) \approx \frac{g_0(l) + g_1(l)s + g_2(l)s^2 \dots}{d_0(l) + d_1(l)s + d_2(l)s^2 \dots} \quad (5)$$

Especially for long TL the periodicity of the imaginary part of $e^{-\gamma(s)l}$ can cause problems. When possible it can be advantageous to split the propagation function into two parts [6]:

$$e^{-\gamma(s)l} = e^{-(\alpha(s) + j\beta(s))l} = W_{att}(s) \cdot W_{delay}(s) = W(s) \quad (6)$$

W_{att} describes the attenuation properties and W_{delay} the phase changes.

1) Splitting the Propagation Coefficient Function

The propagation coefficient function (4) contains the frequency dependent skin effect resistance and internal inductance.

With $s \rightarrow j\omega$ we can write:

$$\gamma = \pm j\omega\sqrt{L'C'} \sqrt{\left(1 - j\frac{R'}{\omega L'}\right)\left(1 - j\frac{G'}{\omega C'}\right)} \quad (7)$$

For the second square root in (7) the following approximation can be used:

$$(1+A)^{1/2} \approx 1 + \frac{1}{2}A \text{ with } A \ll 1 \quad (8)$$

This approximation assumes

$$\left| \frac{R_s'(1-j)}{L'\sqrt{\omega}} \right| \ll 1 \quad (9)$$

which is valid for typical bus cable configurations above some MHz. The propagation constant can be written as follows:

$$\gamma \approx \pm j\omega\sqrt{L'C'} \left(1 - j\frac{R'}{2\omega L'}\right) \left(1 - j\frac{G'}{2\omega C'}\right) \quad (10)$$

Splitting (10) in real and imaginary part gives:

$$\gamma = \pm(\alpha + j\beta) \approx \pm \left(\frac{\sqrt{C'}R'}{2\sqrt{L'}} + \frac{\sqrt{L'}G'}{2\sqrt{C'}} + j \left(\omega\sqrt{L'C'} - \frac{R'G'}{4\omega\sqrt{L'C'}} \right) \right) \quad (11)$$

In the following considerations we assume $G'=0$. Due to very good isolation properties of bus cables used in automobiles, this simplification can be justified. Then the phase velocity β and the attenuation α are:

$$\alpha \approx \frac{\sqrt{C'}R(\omega)}{2\sqrt{L'}} \quad \beta \approx \omega\sqrt{L'C'} \quad (12)$$

2) Considering Skin Effect

Conductor losses at higher frequencies are functions of the square root of frequency \sqrt{f} due to the skin effect. The skin effect dependent per-unit-length (PUL) resistance can be written as [1]:

$$R'(\omega) \approx R_s'(1+j)\sqrt{\omega} \quad (13)$$

For α we obtain considering also the static resistance R_{DC} :

$$\alpha \approx \frac{R_{DC} + R_s'(1+j)\sqrt{\omega}}{2\sqrt{\frac{L'}{C'}}} \quad (14)$$

Using $1+j = \sqrt{2}\sqrt{j}$ we obtain for α :

$$\alpha \approx \frac{R_{DC} + R_s'\sqrt{s}\sqrt{2}}{2\sqrt{\frac{L'}{C'}}} \quad (15)$$

The propagation function can be written as follows:

$$\gamma(s) \approx s\sqrt{L'C'} + \frac{R_{DC}\sqrt{C'}}{2\sqrt{L'}} + \frac{R_s'\sqrt{s}\sqrt{C'}}{\sqrt{2}\sqrt{L'}} \quad (16)$$

As described above, it is possible to extract the delay and model the propagation function by a product of delay function and attenuation function:

$$e^{-\gamma(s)l} \approx \underbrace{e^{-s\sqrt{LC}l}}_{W_{delay}} \cdot \underbrace{e^{-\frac{R_{DC}\sqrt{C'}l}{2\sqrt{L'}} - \frac{\sqrt{s}\sqrt{C'}R_s'l}{\sqrt{2}\sqrt{L'}}}}_{W_{att}} \quad (17)$$

In [6] another slightly different derivation can be found.

3) Approximation of the Attenuation Function

To include DC losses a full approximation with a rational function is necessary. The standard Padé approach is used here.

The attenuation function $W_{att} = e^{-\frac{R_{DC}\sqrt{C'}l}{2\sqrt{L'}} - \frac{R_s'\sqrt{C'}l}{\sqrt{2}\sqrt{L'}}\sqrt{s}}$ must be approximated with:

$$W_{att} \approx \frac{g_0 + g_1s + g_2s^2 \dots}{d_0 + d_1s + d_2s^2 \dots} \quad (18)$$

The function is quite smooth and a small number of coefficients are sufficient to approximate the attenuation function. The coefficients can be determined analytically with a computer algebra program. In the VHDL-AMS implementation 4th order models are used. The real and the imaginary part of the approximated and the exact functions are shown in figure 1. It can be seen that the approximation function is in good agreement to the original function.

B. Treatment of the Characteristic Admittance

Assuming $G'=0$ in (3), as stated before, with $R'(\omega) \approx R_s'(1+j)\sqrt{\omega}$ and adding the DC resistance R_{dc} , the following expression can be written for the admittance:

$$Y_0(s) = \sqrt{\frac{sC'}{R_{dc} + R_s'\sqrt{s}\sqrt{2} + sL'}} \quad (19)$$

The PUL-parameter R_s' is geometry dependent and needs to be derived for each individual geometry. We consider a pair of circular wires. In [1] a high frequency approach is given. The PUL-resistance can be written with the conductivity σ as:

$$R_s' = \frac{1}{2\pi r \sqrt{2}} \sqrt{\frac{\mu}{\epsilon}} \quad (20)$$

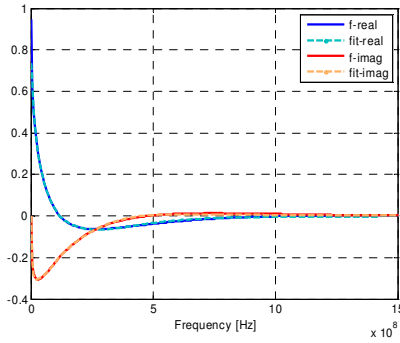


Fig. 1: Real and imaginary part of the approximated (fit) and the exact function (f) of the attenuation with a 4th order approximation

1) Approximation of the Characteristic Admittance Function

The admittance function (19) must also be approximated into a rational function. As frequency extraction is not possible, normalization cannot be done. Nevertheless the function is smooth and a small order of approximation is sufficient to represent it with a Padé approach:

$$Y_0(s) = \sqrt{\frac{sC'}{R_{dc} + R_s' \sqrt{s} \sqrt{2} + sL'}} \approx \frac{n_0 + n_1s + n_2s^2 \dots}{m_0 + m_1s + m_2s^2 \dots} \quad (21)$$

The coefficients can be determined analytically with an symbolic algebra program. In the VHDL-AMS implementation 2nd order models are used. The real and the imaginary part of the approximated and the exact functions are shown in figure 2.

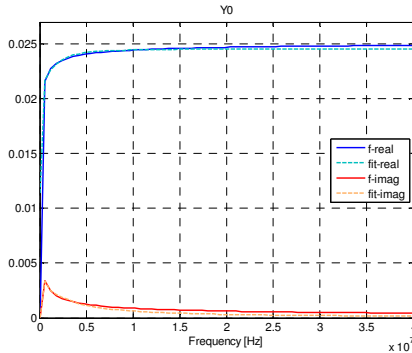


Fig. 2: Real and imaginary part of the approximated (fit) and the exact function (f) of the characteristic admittance with a 2nd order approximation

This second order approximation is quite sensible to the development point of the Taylor series required for Padé approximation. Choosing the development point too low causes the values at high frequencies to differ and vice versa. A higher approximation order would solve the issue but causes convergence problems with the used commercial VHDL-AMS simulators.

III. RESULTS – VERIFICATION OF METHOD

In this chapter the simulation results calculated by the developed transmission line model are presented. In order to

show a practical relevant setup a differential transmission line like a FlexRay cable is investigated. To model this three conductor cable with the effects of even and odd mode transmission the presented two conductor model is used for each mode. The terminal voltages and currents of the even and odd cables (V_{0e} , V_{0o} , I_{0e} , I_{0o}) can be calculated in terms of cable input voltages (V_1 , V_2) and currents (I_1 , I_2) of both cable sets. The even and odd mode systems can be described by:

$$V_{0e} = \frac{1}{2}(V_1 + V_2) \quad I_{0e} = \frac{1}{2}(I_1 + I_2) \quad (22)$$

$$V_{0o} = \frac{1}{2}(V_1 - V_2) \quad I_{0o} = \frac{1}{2}(I_1 - I_2) \quad (23)$$

Assuming symmetric cable sets this approach yields two decoupled cable sets.

In section A a comparison between simulated and measured results is shown. In section B the simulated results of the transmission line model with skin effect are compared to the simulated results of a lossless transmission line model described in [9].

A. Comparison Between Simulation and Measurement Results

To verify the results gained by simulations with the developed transmission line model considering skin effect losses, two different test cases are used. The following sections describe the setup, the measured and the simulated data and a comparison between the latter for both test cases.

Test Case 1

The measurement setup for test case 1 is shown in Fig. 3.

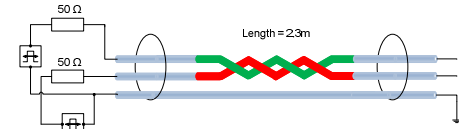


Fig. 3: Measurement setup for test case 1

A FlexRay cable from the manufacturer Gebauer & Griller of type FLM02YHBY with a length of 2.3 m and a conductor radius of 0.35 mm is connected at the near end with a Tektronix pulse generator Type AFG 3252 with 50 Ω internal resistance at each channel.

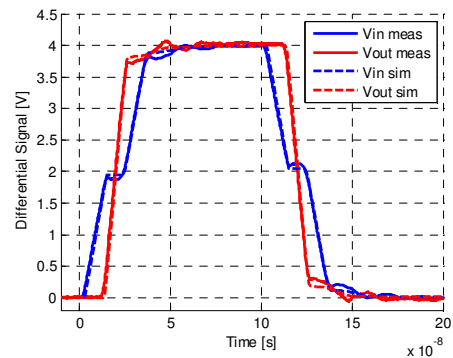


Fig. 4: Measurement and simulation results for test case 1

The far end of each wire is left open. The shield is connected to a metallic ground plan. The differential impedance was estimated for the simulation model with a

simple measurement approach. A value of 90Ω was found. A differential pulse typical for the FlexRay bus system with an amplitude of 2 V and a duration of 100 ns with rise and fall times of 10 ns is fed into the cable. The differential signals at the near and far ends of the line are compared with a Tektronix scope Type TDS 3024B, Fig. 4 shows a good matching between the results.

Test Case 2

A slightly more complex setup is used for test case 2 as shown in Fig. 5.

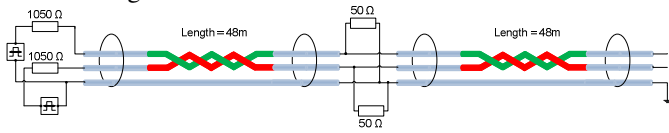


Fig. 5: Measurement setup for test case 2

Two FLM02YHBY FlexRay cables from test case 1 with a length of 48 m each are connected and terminated at the connection point with 50Ω at each line against ground. At the near end of the first line the Tektronix pulse generator is connected with an additional resistor of $1 \text{ k}\Omega$ at each line. The far end of the second cable is left open. A longer pulse with duration of $25 \mu\text{s}$ is fed into the cables. The differential signals at the near end of the first cable, at the far end of the second cable and at the connection point are compared. As in test case 1 a good matching between measurement and simulation results could be achieved (see Fig. 6).

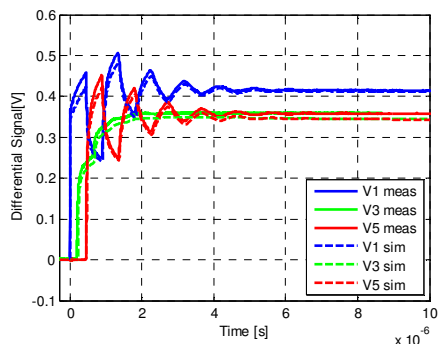


Fig. 6: Measurement and simulation results for test case 2 (V1 are the differential signals at the near end, V3 at the connection point and V5 at the far end of the cables)

B. Comparison Between Lossless and Lossy Transmission Line Models

In this section the simulation results of the lossy multiconductor transmission line model with skin effect are compared to those of a lossless multiconductor transmission line model described in [9]. In the following test case different cable lengths are used within the same test setup: At the near end of each line a pulse generator is connected with its internal resistance of 50Ω . The far end the lines are terminated with an impedance of 50Ω .

Test Case 3

With short transmission lines, the effects of losses can be neglected. Fig. 7 shows that with a line length of 2 m there are

almost no differences between a lossy and a lossless transmission line model. A differential pulse with duration of 100 ns and 10 ns rise time is used with each line.

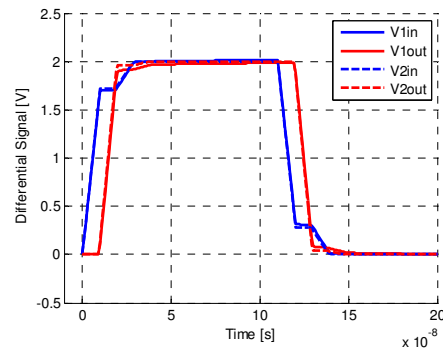


Fig. 7: Comparison between simulation results of lossy and lossless lines with a length of 2 meters each (V1 are the differential signals at the ends of the lossy line and V2 at the ends of the lossless line)

IV. CONCLUSION

A VHDL-AMS model for a three conductor transmission line with frequency dependent losses for transient simulations was developed. VHDL-AMS is a modern simulation language widespread in the industry. The transmission line model can be extended by a large set of available linear and nonlinear models. Validation was done with several test cases comparing simulations and measurements and comparing the described lossy model with a lossless multiconductor model. It could be shown that the model gives accurate results and can be used for practical applications e.g. FlexRay cable simulation. Future work will be focused on determination of accurate transmission line parameter sets and further verification investigations.

REFERENCES

- [1] C. R. Paul, Analysis of Multiconductor Transmission Lines, John Wiley and Sons, 1994
- [2] J.A. Brandao Faria, Multiconductor Transmission-Line Structures: Modal Analysis Techniques, John Wiley and Sons, 1994
- [3] C. R. Paul, A Brief History of Work in Transmission Lines for EMC Applications, IEEE Transactions on Electromagnetic Compatibility, Volume 49, Issue 2, Pages 237 – 252, May 2007
- [4] H. Zhang, K. Siebert, S. Frei, T. Wenzel, W. Mickisch, Multiconductor Transmission Line Modeling with VHDL-AMS for EMC Applications, IEEE International Symposium on EMC, 2008
- [5] D. B. Kuznetsov, J. E. Schutt-Aine, Optimal Transient Simulation of Lossy Lines, IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications, 1996
- [6] K. S. Oh, Accurate Transient Simulation of Transmission Lines with the Skin Effect, IEEE Transaction on Computer-Aided Design of Integrated Circuits and Systems, 2000
- [7] IEEE Standard VHDL Analog and Mixed-Signal Extensions 1076.1, 1999
- [8] R. Achar, M. S. Nakhla, Simulation of High-Speed Interconnects, Proceedings of the IEEE, Vol. 89, No. 5, May 2001
- [9] K. Siebert, H. Zhang, S. Frei, Modellierung der Einstrahlung elektromagnetischer Felder in Mehrleitersysteme mit VHDL-AMS, EMV Düsseldorf 2008