Abstract - Numerical study of the coupling into the metallic enclosure with aperture of transient fields related with indirect Electrostatic Discharge (ESD) is presented. Computer simulation of ESD from spheroid is performed to model human hand discharge. Computed ESD fields are in a good agreement with experimental data. Such realistic fields are used as incident fields to investigate aperture penetration into two-dimensional cavities. Time domain analysis of shielding effectiveness of rectangular cavity with aperture is done using FDTD method.

1. INTRODUCTION

Usually the metallic shields are used to prevent equipment, sensitive electronic parts and systems of which can be disturbed or even destroyed by transient EM fields. The propagation of energy inside the enclosures occurs principally via transmission lines, transmission line-like structures and different kind of apertures, realized for cooling purposes and cable connections.

This paper presents numerical study of the coupling of transient fields radiated during ESD into the metallic enclosure with aperture. Computer simulation of ESD is done using an electrodynamic method based on the Method of Moments in time domain for the discharging bodies of revolution, located near the grounded plane [1-4]. For the discharging structure like spheroid, that models human/hand related ESD, calculated arc currents and fields were compared with experimental data and showed sufficient for EMC applications accuracy of the developed technique [1-4]. Such realistic fields are used as incident fields to investigate aperture penetration into two-dimensional cavities. Time domain analysis of shielding effectiveness of rectangular cavity with aperture is done using FDTD method.

2. MATHEMATICAL MODEL OF ESD

In this Section we describe briefly the mathematical model of ESD.

In the moment before discharge the static charge distribution $\rho_{\text{stat}}(\vec{r})$ is known on all metallic surfaces. During the discharge this distribution will be disturbed. Let us denote this disturbed part of charge by $\rho_{\text{trans}}(\vec{r}, t)$. The whole charge density in any point on the surface of the body or on the plane is $\rho(\vec{r}, t) = \rho_{\text{stat}}(\vec{r}) + \rho_{\text{trans}}(\vec{r}, t)$. The current density $J_{\text{rt}}(\vec{r}, t)$ deals with transient part of charge density. So the transient problem can be stated as the problem of defining the surface current densities $J_{\text{rt}}(\vec{r}, t)$ and arc-current density $J_{\text{arc}}(\vec{r}, t)$. By these quantities all charges and fields can be calculated. Mathematical model for $J(\vec{r}, t)$ and $J_{\text{arc}}(\vec{r}, t)$ can be formulated as follows.

1. On the perfectly conducting surfaces $J_{\text{rt}}(\vec{r}, t)$ satisfies Integral Equation for Magnetic Field (IEMF):

$$J(\vec{r}, t) = \frac{1}{2\pi} \hat{n} \times \vec{H}_{\text{arc}}(\vec{r}, t) + \frac{1}{2\pi} \hat{n} \times \int_{S} \left[ \frac{J(\vec{r}', \tau)}{R} + \frac{1}{c \frac{\partial}{\partial \tau}} J(\vec{r}', \tau) \right] \times \frac{\vec{R}}{R^2} \, ds, \quad \vec{r} \in S$$  \hspace{1cm} (1)

2. In each point of the arc the current density satisfies Ohm’s differential law:
\[ \vec{J}_{\text{arc}}(\vec{r}, t) = \sigma(\vec{r}, t) \cdot \left( \vec{E}_{\text{stat}}(\vec{r}) + \vec{E}_{\text{trans}}^{\text{arc}}(\vec{r}, t) + \vec{E}_{\text{surf}}^{\text{trans}}(\vec{r}, t) \right), \quad \vec{r} \in V_{\text{arc}} \]  

(2)

3. Initial values are written as follows:

\[
\begin{cases} 
\forall \vec{r}, \ t \leq 0: & \vec{J}(\vec{r}, t) = 0; \\
\vec{J}_{\text{arc}}(\vec{r}, t) = 0; \\
\rho_{\text{trans}}(\vec{r}, t) = 0. 
\end{cases}
\]  

(3)

Let us consider terms of these expressions. In the equation (1) \( \vec{H}_{\text{arc}}(\vec{r}, t) \) is magnetic field obtained on the metallic surface. This field is radiated by arc-current. \( S \) is the area of surface of the body and of its image. \( \vec{r} \) is the point of observation, \( \vec{r}' \) is the point of integration, \( \tau \) is the delay time:

\[ \tau = t - R/c; \quad R = |\vec{R}| \] and \( R = \vec{r} - \vec{r}' \). \( c \) is the speed of light, \( \hat{n} \) is the outer normal of the surface. In equation (2) \( \vec{E}_{\text{stat}}(\vec{r}) \) is electrostatic field produced by static charge distribution before discharge. This field initiates the spark. \( \vec{E}_{\text{trans}}^{\text{arc}}(\vec{r}, t) \) and \( \vec{E}_{\text{surf}}^{\text{trans}}(\vec{r}, t) \) are transient fields, radiated by the arc and the surface and their mirrors.

Arc conductivity \( \sigma(\vec{r}, t) \) in equation (2) is function of electric field in the channel. In this work this conductivity is calculated by empiric model of Rompe and Weizel. According this model arc resistance is function of time [6]:

\[ R(t) = \frac{2h}{\sqrt{2aR \int_{0}^{1} t_{\text{arc}}(t') dt'}} \]  

(3)

where \( R(t) \) is arc resistance ([Ohm]), \( 2h \) is length of arc and its image ([m]), \( t_{\text{arc}}(t) \) is arc current ([A]), \( a_{R} \) is empiric constant ([m²/V²sec]), \( t \) is time ([sec]). In expression (3) it is implied that currents along the channel does not vary. That means that the distribution of electric field along the channel implied to be homogeneous.

The equation (1) coupled with nonlinear equation (2) is solved by time domain Method of Moments. Algorithm allows the calculation of transient fields and simulation of whole electrodynamic process of discharge. The results are compared with measurements show good agreement [2-4].

3. FIELDS RADIATED BY ESD

The question how the discharging body radiates into the space is of great importance because the answer gives hints about the region where sensitive apparatus might be disturbed.

In the Fig. 1 the geometry of discharging structure is shown. Metallic spheroid of semi-axes \( a=31 \text{ cm} \) and \( b=5 \text{ cm} \) is chosen in order to model human hand discharge.

Fig. 2 shows the radiated electric field for two different space directions. The angles are calculated in spherical coordinate system.

![Fig. 2. E-field radiated by ESD of 5kV from a spheroid 0.31m and 0.05m semi-axes, arc length \( h=0.6 \text{ mm} \). a) 90° direction; b) 30° direction.](image)

One sees that the smaller the angle becomes, the shorter the pulses become, e.g., they contain more high frequency components. But the maximum value does not change much apart from very small angles. This implies that for apparatus sensitive to high frequency components the most dangerous places do not have to be at a 90 angle (on the plane). Fig. 3 shows normalized spectral contents of electric fields radiated in two particular directions.
Fig. 3. Frequency dependence of the fields shown in Fig. 2. Solid line - 90°; dashed line - 30°. (k=2πf/c)

4. TIME DOMAIN ANALYSIS

In this section cylinders with rectangular cross sections are investigated under illumination of the plane wave having the same time-dependence and magnitude, as a transient field of ESD in some distance near the discharging body.

The method of investigation is Finite-Difference Time-Domain Method (FD-TD) [5]. Let us consider Maxwell’s equations for two-dimensional case (TM polarization).

\[
\begin{align*}
\frac{\partial H_x}{\partial t} & = -\mu \left( \frac{\partial E_y}{\partial y} \right) \\
\frac{\partial H_y}{\partial t} & = -\mu \left( \frac{\partial E_x}{\partial x} \right) \\
\frac{\partial E_z}{\partial t} & = -\frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)
\end{align*}
\]  

(4)

In the FD-TD formulation both space and time are discretized. For the spatial increments Δx and Δy, and the time increment Δt the notation are: subscripts i,j correspond with coordinates ix and jy. Time moment nΔt is denoted by n. So

\[ F^n(i,j) = F(i\Delta x, j\Delta y, n\Delta t). \]

For the derivatives the following approximations are used:

\[
\begin{align*}
\frac{\partial F^n(i,j)}{\partial x} & = \frac{F^n(i + \frac{1}{2}, j) - F^n(i - \frac{1}{2}, j)}{\Delta x} \\
\frac{\partial F^n(i,j)}{\partial t} & = \frac{F^{n+\frac{1}{2}}(i,j) - F^{n-\frac{1}{2}}(i,j)}{\Delta t}
\end{align*}
\]

For Yee’s method of discretization, components of \( \vec{E} \) and \( \vec{H} \) are evaluated at interleaved spatial grid points and interleaved spatial time steps [5]. The spatial grid points in the plane z=0 and the field components evaluated at these points are shown in Fig. 4.

The Maxwell’s equations (4) can be rewritten in the following numerical form:

\[
\begin{align*}
H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}) & = H_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}) \\
& + \frac{\Delta t}{\mu} \left( E_y^n(i, j) - E_y^n(i, j + 1) \right) \\
H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j) & = H_y^{n-\frac{1}{2}}(i + \frac{1}{2}, j) \\
& + \frac{\Delta t}{\mu} \left( E_x^n(i + 1, j) - E_x^n(i, j) \right) \\
E_z^{n+1}(i,j) & = E_z^n(i,j) \\
& + \frac{\Delta t}{\varepsilon} \left( \frac{H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j) - H_y^{n-\frac{1}{2}}(i + \frac{1}{2}, j)}{\Delta x} \\
& - \frac{H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}) - H_x^{n+\frac{1}{2}}(i, j - \frac{1}{2})}{\Delta y} \right)
\end{align*}
\]

(5)

Note that the grid in Fig. 4 is arranged so that the electric field component tangential to the surface of a perfect conductor is evaluated at the surface.

The method described above determines the complete electromagnetic field in the space surrounding the cylinder for all times of interest. This wealth of information can be displayed in a manner to enhance our understanding of the fields coupling phenomena. For this purpose we present the graphs in Fig. 5.

These graphs show the electric field in the space near and inside the cylinder for different moments of time. In Fig. 5, for the moment t=3.75ns the incident pulse just approached to the cylinder and first part of energy began to penetrate the aperture. The next graph shows the cylindrical wave inside the cavity with the center located on the aperture. This field propagates with small reflection from the walls. Reflection can be detected via curvatures in the lines of field near the walls. Graph
on the Fig. 5 for t=6.67ns shows the moment when field reaches the right wall of the cylinder. After this field begins to reflect from the wall and due to the geometry it behaves like the field in the resonator. For the quite a big aperture incident field is penetrating inside and all graphs in Fig. 5 can be explained by consideration of pulse propagation inside the cavity. After a long time we can see resonant fields inside the cavity. These resonant fields can be dangerous for electronic equipment.

To estimate shielding effectiveness we calculate the value

\[ S_{\text{eff}} = \frac{\max E_{\text{total}}}{\max E_{\text{inc}}} \times 100\% \]

In the Fig.6 the shielding effectiveness of rectangular cylinder is shown.

3. ACKNOWLEDGMENT

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4. REFERENCES:


BIOGRAPHICAL NOTES

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